Proposition de sujet (2021-2022)

Kleene Stars in Shuffle and Stuffle Algebras

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As a matter of fact, there is an "Art of Computation" for conc-characters which consists in using an associated comultiplication. This is connected to a lot of geometry of infinite dimensional Lie groups of characters (and their many Lie algebras of primitives) and algorithms to compute identities. Our aim is to explore theory and experiments in

1. Holomorphic Functional Calculus.
2. Systems of Local coordinates (beyond the onces indexed by Lyndon words).
3. Exploration of irreducible points in polynomial rewriting and the graph of dependencies.
4. Algorithmic identification of local coordinates. Last but not least, the potential candidate not only knows combinatorics and category theory but has skills in Python (which is a very natural interface for Sage and other CAS).

This subject is a continuation of [5] where a stable model of "polylogarithmic calculus" (theory of domains) is established. The nuclearity of the target [13] is there to ensure that the algorithmic results actually exist and fit with analysis. We propose here, in particular, a local theory, adapted to a full calculus on indices of Harmonic Sums based on the Taylor expansions, around zero, of polylogarithms with the letter x_1 on the rightmost end. This theory is not only compatible with Stuffle products but also fits with the Analytic Model. In this respect, it provides a stable and fully algorithmic model for Harmonic calculus.

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