PhD title: "An integrated predictive-prescriptive robust optimization approach"

Advisor: Roberto Wolfler Calvo, LIPN (roberto.wolfler@lipn.univ-paris13.fr)
Co-advisor: Roberto Baldacci, Università di Bologna (baldacci@unibo.it)
Co-advisor: Marco Caserta, IE University, Madrid (marco.caserta@ie.edu)

Scope

We consider a decision problem with uncertainty, in which some of the parameters of the problem are generated under an unknown distribution function. This decision problem can be framed under the predictive-prescriptive framework, where one wants to address an estimation problem first (the predictive part) and, subsequently, input such estimates into an optimization model (the prescriptive part of the problem.) With respect to the first part of the decision process, classical approaches to the estimation step rely on the use of machine learning techniques as well as conditional density estimation. On the other hand, robust optimization can be employed as a tool for the optimization under uncertainty. Broadly speaking, robust optimization allows to enrich the aforementioned framework since, rather than optimizing over a single point estimate of the uncertain parameter, one optimizes over what is known as an uncertainty set. However, it is well known that the classical robust optimization approach, whose aim is to immunize against the worst case scenario, tends to produce overly-conservative solutions. With this in mind, we are interested in exploring alternative approaches (both on the predictive and the prescriptive side of the decision problem) which mitigate such effect, while still retaining a high degree of immunization. The predictive-prescriptive framework can be tackled from two different perspectives:

- Using the standard stochastic approach, i.e., separating the predictive and the prescriptive problems and, therefore, defining a two-phase approach. Under this scenario, the results of the predictive models are provided as input to the robust optimization phase. A number of robustness paradigms, e.g, Distributionally Robust Optimization, multi-stage Robust Optimization, can be explored to address the optimization phase.
- Integrating the predictive and the prescriptive portions of the decision problem. That is, one could envision the creation of a prescriptive model that encompasses elements of the predictive model within the optimization phase.

Research proposal

This proposal aims at integrating predictive and prescriptive models within a robust optimization framework. The proposal will validated the approach presented in the introduction using as case study the management of the operating rooms system. Indeed, one of the critical activities of an hospital planner concerns the scheduling of the operating rooms (For example, Bandi and Gupta [1] state that operating rooms generate up to 70% of a private hospital revenue.) This issue has attracted significant scholars' attention since surgical operations account for around 40% of the total hospital cost ([3]). The challenge behind the scheduling is connected with the unpredictability and high variability of the duration of the planned surgeries which, in turn, contributes to large delays in the execution of the surgeries and/or costly gaps in the planning. With this in mind, we can divide the proposal in two steps:

- **Step 1: Surgery Duration Characterization**
  Firstly, we want to build a data-driven predictor using, e.g., machine learning, to estimate the duration of a planned surgery, along with some measures of its expected variability.
- **Step 2: Robust Scheduling Problem**
  Secondly, we want to define an appropriate mathematical model to find the optimal scheduling of operating rooms. Subsequently, the analysis performed in the first step will be used to capture the different scenarios arising in nature and will be used to find a robust schedule of the operating theaters.

Both steps are of paramount importance with respect to the quality of the overall decision. The connection between the accuracy of the prediction and the value of an upper bound of the robust solution has been established in the literature. For example, in [10] the authors show that the more accurate the prediction of the ML model is, the less conservative we need to be once the uncertainty is embedded into the model. As discussed in the literature, there is a direct relationship between the quality of the solution of the scheduling problem and the quality of the predictive framework. Therefore, developing accurate predictive models to estimate the duration of surgery and effective models is in general is equally important.

Within each step, we will try to address a few fundamental Research Questions (RQs). For simplicity of presentation, the various questions are presented as separate. However, their interdependence is clear and they should not be seen as separate problems but rather as different aspects that at the end are combined to achieve a unique goal.

Step 1 - RQ1: Is it possible to use feature engineering to increase the knowledge about the data?

To further enrich the dataset, a number of constructed variables can be built for each observation. For example, we could compute measures of team familiarity, i.e., how often the given team has worked together, leader familiarity, i.e., how often the main surgeon has worked with the team, or experience diversity of the team, i.e., how often these members of the team have been exposed to other people. The aforementioned indicators can be then used within a machine learning algorithm to come up with a forecast of a given surgery duration.

Step 1 - RQ2: What is the most accurate forecasting technique to estimate duration of surgeries

The goal is to create an algorithm to forecast the duration of a surgery, given a set of characteristics (service, OMD code, team, etc). We want to compare two different perspectives, namely machine learning techniques as well as density-based methods. From the perspective of the machine learning models, we plan to use the standard global models (i.e., models in which the entire dataset is considered) and specialty-specific models (i.e., a group of predictive models for each specialty) based on the use of Ridge Regression, Random Forest Regressor, Support Vector Regressor and Gradient Boosting-based methods. As second set of methods, we plan to use statistical methods based on Conditional Density Estimation (CDE) to forecast the duration of surgeries. CDE is concerned with the task of inferring conditional probability distribution functions for the surgery durations, where the condition is dictated by the input parameters. Our focus in this second case will not only be on creating a model which produces the best possible forecast, but we will try to look at the problem from a broader perspective. As mentioned in the introduction, the motivation behind this study is to create a robust and reliable scheduling of the operating theaters. Such robustness depends on the ability to create accurate forecasts for the duration of the different procedures. The machine learning methods presented can achieve such goal. However, in the context of building robust models for the scheduling problem, one might be interested in constructing a probability distribution function conditioned on the input parameters. As pointed out in [5], regression can be seen as a CDE problem, although the emphasis is on the mapping of the input onto the dependent variable, rather than on the construction of a probability density function.

Step 2 - RQ1: What is the Scheduling Problem that better reflects the two case studies considered?

The scheduling of operating rooms has attracted a great deal of attention. As pointed out by May et al. [7], a large portion of an hospital expenditure accrues to the management of operating rooms. We direct the interested readers to May et al. [7], Zhu et al. [11] for a literature review of the different problems concerning the optimal management of the ORs. According to the taxonomy presented in May et al. [7], the surgical scheduling problem is broken down into six different problems depending on the considered time horizon, i.e., capacity planning, process reengineering, surgical services portfolio, procedure duration estimation, schedule construction, and implementation. In this proposal, we will focus our attention on the schedule construction problem, which is considered a short-term problem. Typically, the schedule construction problem takes place between a few days and a few weeks before the focal time period, and is concerned with the assignment of procedures to a set of ORs, while satisfying constraints such as surgeon availability, room preferences and limitations, and other side constraints. The problem has been approached from the scheduling perspective, as well as a bin packing problem. In addition, a number of simulation studies have been proposed. Finding a good mathematical model to describe all the decision variables involved is therefore not an easy task. In our case, the mathematical model must provide a planning with an horizon of one week, i.e., at the end of each week the planner generates a (static) plan for the incoming week. Each surgery time is characterized by a duration, which corresponds to the total time during which the operating theater will be occupied. The total surgery time is obtained as the sum of a number of activities. Each surgery is associated to a surgeon, while the remaining members of the team are assigned from a pool. therefore, the model will provide both the scheduling of each surgeon for the whole weeks and the compositions of the team used in each operation.

Step 2 - RQ2: How should the uncertainty of the data be integrated in the model?

The goal of this section is to integrate the results obtained in Step 1 with the model obtained in Step 2. Basically, we could either approach them sequentially (first the forecast of the times and the construction of the uncertainty set and, next, the optimization under the robust paradigm), or simultaneously (the integrated model includes both the forecast and the robust optimization). Zhu et al. [11] present an extensive literature review, in which over 300 papers are mapped along six different dimensions (decision level, scheduling strategy, patient characteristics, problem features, mathematical model, and solution approach.) Of special interest is the characterization of the problem feature related to the uncertainty in the duration of the procedure. From the extant literature, it emerges that the uncertainty of the duration of surgeries has been captured using probability distribution functions (typically, the log-normal, gamma, or normal distributions) or uncertainty sets (as in robust optimization, both via two-stage and distributionally robust optimization.). A third, less frequent approach, to deal with uncertainty is related to the use of chance constraints. The starting references we will use to carry out our analysis are: Data-driven robust optimization: Using statistical hypothesis testing and bootstrapping, the uncertainty set is directly built from the data. See, e.g., Bertsimas et al. [2], Dokka and Goerigk [4]. An integrated approach: Find a solution $\mathbf{x}^*$ which remains feasible for the largest possible uncertainty set. See, e.g., Zhu et al. [12]. Use of machine learning, e.g., clustering, to define ambiguity sets to model the uncertain parameters, in line with Distributionally Robust Optimization. For example, in Perakis et al. [8] they create clusters and use the mean and standard deviation of these clusters to define sets within which the robust parameters will change.

Step2 - RQ3: What is the best solution method to efficiently solve the provided models?

The models introduced in the previous RQs will be significantly complex and the size of the real-world instances will be huge. This implies that a mathematical programming solver will be unable to solve them in a reasonable time and/or with an acceptable quality. Therefore, the aim of this RQ is to define efficient heuristic and exact algorithms to solve the models introduced in the previous questions. We propose to use decomposition-based reformulation techniques like column generation [6] and/or Benders decomposition [9].

Real-world Data

To validate the developed methodologies, we plan to test the framework on two sets of data kindly provided by two European hospitals: the "Hospital Universitario Moncloa" in Madrid, Spain, and the "Giannina Gaslini" hospital in Genoa, Italy. Both hospitals are characterized by a significantly high number of operating rooms, each with its own characteristics. The fact of working in collaboration with two important hospitals brings the following advantages: in the first place, it allows to receive feedback on the salient modeling aspects to be considered in the drafting of the optimization model. Secondly, collaboration with hospitals makes it possible to have an already substantial database available. This last aspect is by no means negligible: at the moment, both hospitals are willing to share over 80 thousands observations of scheduled surgeries and their duration, over the a period of two years. The presence of such a database is a good starting point and it allows to focus on the use of the data, rather than on their gathering and preparation.

Required skills

The PhD student should have a master degree in computer science. A background in combinatorial optimization or operational research will be much appreciated.

References

[1] C. Bandi and F. Gupta. Operating room staffing and scheduling. *Manufacturing & Service Operations Management*, 22(5):958–974, 2020.

[2] D. Bertsimas, V. Gupta, and N. Kallus. Data-driven robust optimization. *Mathematical Programming*, 167:235–292, 2018.

[3] Thomas Reiten Bovim, Marielle Christiansen, Anders N Gullhav, Troels Martin Range, and Lars Hellemo. Stochastic master surgery scheduling. *European Journal of Operational Research*, 285(2): 695–711, 2020.

[4] Trivikram Dokka and Marc Goerigk. An experimental comparison of uncertainty sets for robust shortest path problems, 2017.

[5] Vincent Dutordoir, Hugh Salimbeni, Marc Deisenroth, and James Hensman. Gaussian process conditional density estimation. *arXiv preprint arXiv:1810.12750*, 2018.

[6] Hongying Fei, Chengbin Chu, and Nadine Meskens. Solving a tactical operating room planning problem by a column-generation-based heuristic procedure with four criteria. *Annals of Operations Research*, 166(1):91–108, 2009.

[7] J. H. May, W. E. Spangler, D. P. Strum, and L. G. Vargas. The surgical scheduling problem: Current research and future opportunities. *Production and Operations Management*, 20(3):392–405, 2011.

[8] G. Perakis, M. Sim, Q. Tang, and P. Xiong. Joint pricing and production: A fusion of machine learning and robust optimization, 2018.

[9] Vahid Roshanaei, Curtiss Luong, Dionne M Aleman, and David Urbach. Propagating logic-based benders' decomposition approaches for distributed operating room scheduling. *European Journal of Operational Research*, 257(2):439–455, 2017.

[10] Y. Wang, Y. Zhang, and J. F. Tang. A distributionally robust optimization approach for surgery block allocation. *European Journal of Operational Research*, 273(2):740–753, 2019.

[11] S. W. Zhu, W. J. Fan, S. L. Yang, J. Pei, and P. M. Pardalos. Operating room planning and surgical case scheduling: a review of literature. *Journal of Combinatorial Optimization*, 37(3):757–805, 2019.

[12] T. Zhu, J. Xie, and M. Sim. Joint estimation and robustness optimization, 2019.