Contrat Doctoral — ED Galilée

Titre du sujet : Phase-space analysis of self-interacting Quantum Field Theories.

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- ➢ Discipline : Mathématiques.
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- ➢ Mots-clés : Microlocal and semi-classical analysis, quantum field theory, spectral analysis.

Objectif de la thèse : Quantum field theory is one of the topics of mathematical physics that raises the most prominent challenges to mathematics, see [Jaf00, JW06]. On the other hand the subject is experiencing a renewed interest, particularly in relation to the recent trends of employing probabilistic tools and stochastic quantization to the analysis of nonlinear PDEs. In this context, we aim to introduce a new approach to QFT based on phase-space analysis in infinite dimensional spaces and to show that this point of view leads to a better understanding of some non-perturbative aspects of quantum field theories. The old results of the sixties and seventies provide us for instance with elegant constructions of the φ^2, φ^4 and φ^{2n} non-linear quantum fields in two and three-dimensional space-times. Moreover, pseudo-differential operators in infinite-dimensional spaces have been introduced and studied since at least the nineties, see for instance the work of Sergio Albeverio and Alexei Daletskii [AD98]. In the recent years, it has become clear that several ideas inspired by micolocal analysis are quite fruitful in quantum mechanics, in electrodynamics and in many-body theory, see [AZ14, AN08, AJN15, CR21, BFRZ14, FZ04, WZ12] and [WZ21]. We therefore propose in this thesis a formal extension of microlocal analysis to infinite dimensional phase spaces and a study of the φ^2, φ^4 and φ^{2n} quantum field models with these new tools. Complete asymptotic of coherent states, propagation of singularities and semi-classical expansions of ground states energies would be among the main issues to be addressed.

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