MONARC:
MODELING OF GROUNDWATER-RIVER EXCHANGES FOR CLIMATE CHANGE PREDICTIONS

1 Description of the project

The MONARC project aims to model all hydraulic processes, both surface and underground, on the scale of a watershed and for periods of several decades. This modeling seems essential to produce numerical calculation tools that can be used for regional forecasts in a context of climate change, particularly for water resource and morphodynamic issues. It is mainly based on vertically integrated models, typically the Saint-Venant and Dupuit-Forchheimer equations. This type of model does not easily take into account the infiltration processes. The problem of infiltration into the unsaturated zone, known as the vadose zone, seems to be the most serious obstacle to accurate predictions. This problem is at the heart of the Monarc project.

Infiltration and changes in the water table and streams play an important role in the hydraulic response of a watershed [3]. Two types of infiltration regime can be distinguished. Rainwater infiltration can be very high when rainfall is moderate, contributing to the effective recharge of the water table, until the porous medium is completely saturated, which can lead to flooding due to the emergence or rising of the water table. Conversely, when rainfall is intense, the surface soil rapidly becomes saturated and the water no longer infiltrates (capping phenomenon), leading to a preponderance of run-off accompanied by rapid, localised flooding.

Thanks to the initial work carried out as part of the ANR GeoFun project (2020-2024), we now have a model capable of a faithful representation of the processes of emergence and rising of the water table, i.e. exchanges from the porous medium to the surface [2]. Infiltration processes are totally neglected as the medium is assumed to be totally saturated from bottom to top. More explicitly, the water falls directly to the bottom of the aquifer to be added to the water table. This assumption leads to an underestimation of the response time of the medium in the case of slow infiltration, and to an inability to represent runoff on an unsaturated medium. In order to remedy this shortcoming, we propose to proceed in three steps:

1.a Vertical infiltration without mesh : Infiltration in the vadose zone is usually modelled using the Richards model. This is a 3D model that describes the flow in the vadose zone (unsaturated zone) as well as in the water table (saturated zone). It is written as

$$\partial_t s - \nabla \cdot (\kappa s \nabla (p + gz)) = 0 \tag{R}$$

with κ the conductivity and g the gravity acceleration. The unknowns are the saturation s and the pressure p. The pressure is activated in the water table so that the saturation cannot exceed a maximum threshold $\overline{s}$ corresponding to the porosity, i.e.

$$\min(\overline{s} - s, p) = 0 . \tag{C}$$

Looking at the model only in the vadose zone, it reduces to a simple vertical transport equation, whose analytical solution can be explained for relatively simple conductivities. However, this equation is only valid in the vadose zone whose lower limit corresponds to the piezometric level and is therefore an unknown of the problem. In this first step, we propose to focus on the resolution of this unidirectional transport equation coupled with a variable boundary condition. A strategy based on a layer approach will be proposed to take into account the discontinuities of the conductivity and porosity. In each column, we will consider the equation

$$\partial_t s - \partial_z (g\kappa s) = G \qquad \text{with each } z < \eta(t) \qquad \min(G(t,z), s(t,z)) = 0$$

where $\eta(t)$ is the piezometric level (unknown of the problem), whose evolution depends in particular on the contribution by the vadose zone. In a simple case without horizontal exchange, we have

$$\partial_t (V(\eta(t))) = \int_{-\infty}^{\eta(t)} G(t,z)\,\mathrm{d}z \qquad \text{where} \qquad V(\eta) = \int_S^{\eta} \overline{s}\mathrm{d}z ,$$

$V(\eta)$ being the volume of water contained in the water table. Considering constant conductivities per layer and a constant upper boundary condition per piece in time (presence or not of liquid in the saturated zone of the layer above), the solution is constant per piece. Thus, a method without mesh (front-tracking) will be considered so that the global model does not depend on an additional discretization parameter.

1.b Coupling with the Saint-Venant/Dupuit-Forchheimer unified model : Once the first step is completed, we will focus on the coupling with the Saint-Venant/Dupuit-Forchheimer model which currently neglects the vadose zone [2]. In this model, the input term G, or exchange between layers since it is not necessarily signed in this case, is already present but the infiltration is assumed to be instantaneous. The coupling with the vertical transport equations introduces a delay term that enriches the physics. The difficulty of the coupling lies in the transitions between the seepage regions $G(t,x) > 0$ and the resurgence regions $G(t,x) < 0$. A detailed analysis of the model will be necessary. The numerical strategy to realize this coupling is not simple either because the number of unknowns is not the same in both cases. Another point to study will be the presence of impermeable zone which may lead to the need to consider a congested model. The mathematical analysis of this situation is very complex for the surface model (Saint-Venant equations) but the greater regularity of the solutions in the underground part (Dupuit-Forchheimer model) offers a more favorable framework.

1.c Numerical validations : The last step focuses on the numerical validation of the modeling and the numerical strategy. For some simple configurations, it is possible to have an analytical solution of the problem, especially for the first step of vertical transport. However, the general framework can only be approximated by a numerical method. We therefore propose to compare the results of the Saint-Venant/Dupuit-Forchheimer model with infiltration with a resolution of a high-fidelity model. Initially, we will focus on the solution in the porous medium and thus the Richards model (R) to be used to validate the approach. The runoff cases require to consider a more general model valid for subsurface flows as well as for surface flows. We will then consider an Euler model with a drag force, i.e.

$$\begin{aligned} \partial_t s + \nabla \cdot (su) &= 0 \\ s\partial_t u + su \cdot \nabla u &= -s\left(\nabla(p - gz) + \frac{u}{\kappa}\right) \end{aligned} \tag{E}$$

where the velocity u is an unknown with s and p. The pressure p is defined by the constraint (C) as for the Richards model (R). In the regime where the conductivity is very low, the Richards equation (R) is formally found, so a *Asymptotic preserving* scheme will be used to preserve this property [1]. Also, simulations approximating the two-phase Navier-Stokes equations on a permeable sandy slope will be carried out using the `Notus` software developed at I2M to validate the model.

Finally, a last validation step will be performed by comparing with an operational code and on a real case with in situ observations. The Somme watershed seems to be an ideal case to demonstrate the relevance of the approach developed in the Monarc project. The test case data and implementation will be facilitated by collaboration with IFPEN hydrologists with whom we are in contact.

2 People involved

The thesis constituting the MONARC project will be directed by Emmanuel Audusse (USPN, LAGA - audusse@math.univ-paris13.fr), Martin Parisot (inria - Bordeaux, EPI CARDAMOM - martin.parisot@inria.fr) and Mathieu Coquerelle (I2M, Bordeaux - mathieu.coquerelle@bordeaux-inp.fr). It should start in autumn 2024. The project is already partially supported by AAP-IMPT[1], which has committed 70,000 euros (see the attached letter). We are applying for the remainder.

References

[1] BULTEAU, S., BADSI, M., BERTHON, C., AND BESSEMOULIN-CHATARD, M. A fully well-balanced and asymptotic preserving scheme for the shallow-water equations with a generalized manning friction source term. *Calcolo 58*, 4 (Sep 2021), 41. <hal-02479094>

[1] Institut des Mathématiques pour la Planète Terre https://aap-impt2023.sciencesconf.org/

[2] CARREAU M. AND PARISOT M. A unified modeling of underground-surface hydraulic processes: case of the saturated regions. *Work in progress*. <hal-03836565>

[3] ERSOY M., LAKKIS O. AND TOWNSEND P.. A Saint-Venant Model for Overland Flows with Precipitation and Recharge. *Math. and Comput. Appl., 2021*. <DOI:10.3390/mca26010001>