PhD dissertation subject

Decomposition methods for quantum quadratic optimization

Context

The aim of this field is to propose new solution methods for quantum quadratic optimization. These problems can be found in many application domains [2, 5], such as energy, finance, logistics, networks, image processing and computer vision. These problems are generally NP-hard and solving them exactly, or even obtaining good relaxations, remains a challenge today, especially for instances of realistic size.

Our goal here is to use and generalize quantum optimization algorithms to speed-up the resolution of decomposition methods for binary quadratic problems.

Quantum computing and binary quadratic optimization

There exist several heuristic quantum algorithms for solving Quadratic Unconstrained Binary Optimization (QUBO) problems [4]. The first digital annealers quantum approaches are able to solve heuristically QUBO of small size, compared to classical heuristics and metaheuristics methods, or even exact approaches. Recent advances in quantum research [6] would allow to scale and solve heuristically larger instances. Researchers also started to consider the linearly constrained case [2, 3], based on Frank-Wolfe method or Lagrangian dual relaxation. This opens the road to address more complex problems.

Decomposition methods for binary quadratic optimization

There exist various decompositions in the scientific literature adapted to particular cases of the binary quadratic problems that will be considered in this subject, such as, for example, the Danzig-Wolfe decomposition, the Benders decomposition, the Lagrangian decomposition, the clique decomposition, the simplicial decomposition, etc.

As a first step, we would like to consider the BQP (Boolean Quadric Polytope) decomposition recently proposed in [1]. This decomposition involves QUBO subproblems to be solve at each iteration of the method. It is in this regard that we would like to take advantage of quantum computing to speed-up the resolution. Preliminary experiments have been performed in collaboration with Sota Hirama who is currently a PhD student at Tohoku University [7]. During Summer 2024, he visited LIPN where he worked with the two prospective advisors in the context of a one-month research stay. We remarked that blindly applying standard quantum annealing techniques is not enough to beat the state of the art non-quantum approaches. In particular, we observed that the feasible solutions given by quantum methods are pretty far from the optimal solutions. This is why we would like to investigate other quantum methods [3] to evaluate the quality of their output.

A second step will be to consider other decomposition techniques, always focusing on the linearly constrained binary quadratic case. Examples of classic decompositions in the context of discrete optimization are Danzig-Wolfe and simplicial decompositions among others. The impact of distinct decomposition approaches will be explored on different applications.

A third step consists of extending the quantum algorithms to deal with quadratically constrained binary quadratic problems. An approach using the Lagrangian relaxation has been mentioned in [3]. A thorough study of this more general case is however lacking in the literature.

Advisors

Advisor: Lucas Létocart, PU, LIPN

Co-advisor: Silvia Di Gregorio, MCF, LIPN

The two co-supervisors have complementary expertise in mathematical optimization, particularly in nonlinear optimization as well as in decomposition methods.

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