

Titre du sujet : Sur les fibrés en vecteurs de Witt et leurs applications en théorie des représentations Galoisiennes

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Context. In the recent years, I developed with M. Florence a geometrization of Kummer theory through the theory of smooth profinite groups ([1], [2], [3]). It aims at revisiting classical problems on Galois cohomology and Galois representations to more general profinite groups, providing a framework connected with inverse Galois theory. Fundamental such problems are related to liftings of algebraic objects from positive characteristic to characteristic zero, as well as the description of the spaces classifying these liftings (geometry of deformation spaces, nature of obstructions, etc.). Notably, looking at liftings of vector bundles to Witt vector bundles in [4], we established a canonical resolution for this deformation problem for line bundles, which can be naturally lifted through the Teichmüller representative, and solved the lifting problem of tautological bundles for Grassmannians.

Objectives. The objective of this thesis is apply this theory in a more arithmetic context. To this end, the candidate will benefit from my expertise and that of Ariane Mézard in the p -adic local Langlands program, and more specifically in the deformations of p -adic Galois representations. The relations defining the Galois deformation ring should indeed in this context be expressed through the more rigid notion of Kummer flags (in the sense of [6]).

First project. As an initial project, the candidate will aim to establish the non-liftability of tangent bundles to Witt vector bundles over flag varieties. This will be achieved using the techniques of [4] as in this context tangent bundles and tautological bundles are closely related. Computing cohomology of line bundles over the flag varieties and using dévissage, the results sought for is that for $n \geq 4$, the tangent bundle TFL_n over a flag variety FL_n does not lift to a W_2 -bundle.

Large Witt vectors and transition between characteristics. The candidate will also be expected to develop this approach in the context of large Witt vectors in order to understand the transition from one characteristic to another. One central question is determining how p -adic and l -adic information complement each other and suffice to yield global properties. This is a crucial question in the study of Galois representations of number fields within the Langlands program, in order to understand the local to global compatibility as well as for function fields. For example, we can cite the recent result by S. Tsujimura : if X is a hyperbolic curve over K (a finite extension of \mathbf{Q}_p), the Galois group G_K injects into the maximal pro- S quotient of the fundamental group of X if S contains the places above p and another auxiliary prime.

Obstructions and deformation theory. In [5], M. Florence establishes a criterion guaranteeing the existence of a smooth projective F -variety X for which an abelian variety A over F is isomorphic to the automorphism group scheme of X/F . The candidate will be invited to revisit this approach within the framework of representation deformation theory to better understand the nature of the obstructions (Massey-Lie classes). The recent article by B. Le Hung, A. Mézard and S. Morra [7] provides a source of examples of deformation rings with non-trivial relations which ought to be explained.

Références

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