PhD dissertation subject

Quantum optimization: from algorithms design to hardware implementation

Context

Quantum computing is increasingly being explored by the optimization community to address hard combinatorial optimization problems. Currently, most of them are unconstrained binary optimization problems, representing a small part of real-world problems (e.g. energy, logistics, etc.). The goal of this thesis is twofold. On the one hand, it aims at designing new quantum algorithms to tackle constraints in combinatorial optimization problems. On the other hand, it seeks at implementing such algorithms into specific hardware, facing many optimization challenges such as mapping and routing qubits.

Quantum algorithms for combinatorial constrained problems

The main problems tackled currently by quantum algorithms are Quadratic Unconstrained Binary Optimization (QUBO) problems using quantum heuristics (Quantum Approximate Optimization Algorithm (QAOA) [1], more general variational quantum methods [2, 3], Quantum Annealing [4] ...). The first approach of this thesis relates to constrained problems. They represent a significant part of everyday challenges, and yet are under-studied from a quantum algorithmic perspective. The classical Lagrangian relaxation, or similar methods, coupled with quantum heuristics for unconstrained problems are mainly invoked for such constrained problems [5], in parallel of reformulations of the initial problem [6]. However, other approaches dealing with constraints at the core of the algorithm exist, such as the Quantum Alternating Operator Ansatz algorithm [7] where constraints are characterized by mixers (Mixer-QAOA). Such mixers have been found for specific constraints (e.g. constant Hamming weight [8, 9]) but building mixers for other general constraints remains an open question.

As a first step, we will adapt such algorithms to solve the Quadratic Knapsack Problem with cardinality constraint (kQKP) [10] by designing mixers for two types of constraints (capacity and cardinality). As a second step, we will study more generic problems, where many variations are possible (in terms of objective function, types of contraints, domains of the variables). Notice that we will also be able to investigate other methods to solve the kQKP by integrating Mixer-QAOA in classical decomposition methods [11].

Optimization and implementation of quantum circuits

As interest in quantum algorithms grows, along with the expansion of quantum hardware, implementing quantum circuits on actual quantum devices has become increasingly important. The second approach of the thesis focuses on an implementation known as *transpilation*, which involves developing classical optimization methods for both the expression of quantum circuits [12] and their mapping (representing qubit assignment plus qubit routing) to a specific quantum chip [13, 14].

In the literature, these several problems are often tackled separately, because already NP-hard. In this thesis, we aim at solving them with an integrated approach, motivated by finding better solutions for the overall solution. Specifically, we will apply them to the specific algorithms designed in the first approach of the thesis, in order to solve more efficiently constrained nonlinear optimization problems with specific quantum optimization algorithms. Moreover, we will add the following new consideration for this problem: scheduling the execution of different circuits, with precedence constraints, on the hardware in order to deal with parallel quantum computing.

Supervision

Co-director:	Lucas Létocart	PU	LIPN, Université Sorbonne Paris Nord
Co-director:	Roberto Wolfler Calvo	PU	LIPN, Université Sorbonne Paris Nord
Supervisor:	Camille Grange	MCF	LIPN, Université Sorbonne Paris Nord

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